

**MARK SCHEME for the May/June 2014 series**

**4037 ADDITIONAL MATHEMATICS**

**4037/21**

Paper 2, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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1	$x^2 + x \geq 0$ critical values 0 and $-1$ soi $-1 < x < 0$	<b>M1</b> <b>A1</b> <b>A1</b>	expands and rearranges  condone space, comma, “and” but not “or” Mark final answer.
2	$\frac{6}{(1+\sqrt{3})^2}$ or $6 = (a+b\sqrt{3})(1+\sqrt{3})^2$  $\frac{6}{4+2\sqrt{3}}$ or $6 = (a+b\sqrt{3})(4+2\sqrt{3})$ $\frac{6}{4+2\sqrt{3}} \times \frac{4-2\sqrt{3}}{4-2\sqrt{3}}$ AND attempting to multiply out $6 - 3\sqrt{3}$ isw	<b>M1</b>  <b>M1</b>  <b>M1</b>  <b>A1</b>	for dealing with the negative index (condone treating 6 as have negative index at this stage)  for squaring  for rationalising or for obtaining a pair of simultaneous equations $4a + 6b = 6$ and $2a + 4b = 0$
3	<p>(i)</p> <p>(ii)</p> $x = 1$ (only) soi $y = \pm 9$ (only) $0 < k < 9$	<b>B1</b> <b>B1</b>     <b>B1</b> <b>B1</b> <b>B1</b>	correct shape $x$ intercepts marked or implied by tick marks, for example or seen nearby; condone $y$ intercept omitted      can be implied by second <b>B1</b> or $k = \pm 9, +9$ or $-9$ or both; must be strict inequality in $k$ ; condone space, comma, “and”, “or”
4	Attempt to find $f(4)$ or $f(1)$ or division to a remainder $128 + 16a + 4b + 12 = 0$ or better $(16a + 4b = -140)$  $2 + a + b + 12 = -12$ or better $(a + b = -26)$  Solves linear equations in $a$ and $b$  $a = -3, b = -23$	<b>M1</b>  <b>A1</b>   <b>A1</b>   <b>M1</b>   <b>A1</b>	condone one error         both

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5	(i)	$2\left(x - \frac{1}{4}\right)^2 + \frac{47}{8} (5.875)$ isw	<b>B3,2,1,0</b>	one mark for each of $p, q, r$ correct; allow correct equivalent values. If <b>B0</b> , then <b>SC2</b> for $2\left(x - \frac{1}{4}\right) + \frac{47}{8}$ , or <b>SC1</b> for correct values but incorrect format
	(ii)	$\frac{47}{8}$ is min value when $x = \frac{1}{4}$	<b>B1ft + B1ft</b>	strict <b>ft</b> <i>their</i> $\frac{47}{8}$ and <i>their</i> $\frac{1}{4}$ ; each value must be correctly attributed; condone $y = \frac{47}{8}$ for <b>B1</b> , or $\left(\frac{1}{4}, \frac{47}{8}\right)$ for <b>B1B1</b>
6	(a)	${}^8C_3 \times 3^3 \times (\pm 2)^5$ or $3^8 \left[ {}^8C_3 \left(\pm \frac{2}{3}\right)^5 \right]$ -48384	<b>M1</b> <b>A1</b>	condone ${}^8C_5, -2x^5$ can be in expansion
	(b) (i)	$1 + 12x + 60x^2$	<b>B2,1,0</b>	ignore additional terms. If <b>B0</b> , allow <b>M1</b> for 3 correct unsimplified terms
	(ii)	Coefficient of $x$ correct or correct <b>ft</b> $(12+a)$ soi Coefficient of $x^2$ correct or correct <b>ft</b> $(60+12a)$ soi $1.5 \times \text{their}(12 + a) = \text{their}(60 + 12a)$ - 4	<b>B1ft</b> <b>B1ft</b> <b>M1</b> <b>A1</b>	<b>ft</b> <i>their</i> $1 + 12x + 60x^2$ <b>ft</b> <i>their</i> $1 + 12x + 60x^2$ no $x$ or $x^2$
7	(i)	$-\frac{1}{x^2} + \frac{1}{x^{1/2}}$	<b>B1 + B1</b>	or equivalent with negative indices
	(ii)	$\frac{2}{x^3} - \frac{1}{2x^{3/2}}$	<b>B1ft + B1ft</b>	or equivalent with negative indices. Strict <b>ft</b>
	(iii)	Attempting to solve <i>their</i> $\frac{dy}{dx} = 0$ $x = 1 \quad y = 3$ Substitute <i>their</i> $x = 1$ into <i>their</i> $\frac{d^2y}{dx^2}$ ; or examines $\frac{dy}{dx}$ or $y$ on both sides of <i>their</i> $x = 1$ Complete and correct determination of nature. If correct, minimum.	<b>M1</b> <b>A1</b> <b>M1</b> <b>A1</b>	must achieve $x = \dots$ (allow slips) <b>SC2</b> for $(1, 3)$ stated, nfw for using <i>their</i> value from $\frac{dy}{dx} = 0$ must be from correct work

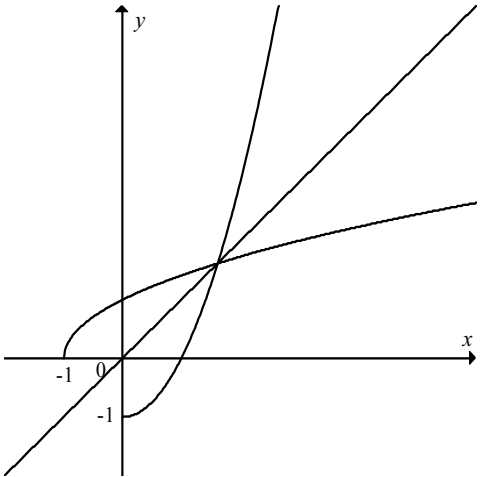
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8	(i)	$2r + r\theta = 30$ giving $\theta = \frac{30 - 2r}{r}$ Substitute <i>their</i> expression for $\theta$ into $A = \frac{1}{2}r^2\theta$ Correct simplification to $A = 15r - r^2$ AG	M1 M1 A1	correct arc formula + (2)r rearranged  $\text{their } \frac{dA}{dr} = 0$  56.3 is <b>A0</b> unless 56.25 seen; if <b>M0</b> , then <b>SC2</b> for $A = 56.25$ with no working; or <b>SC1</b> for $r = 7.5$ with no working
	(ii)	$15 - 2r = 0$ $r = 7.5$ 56.25	M1 A1 A1	
9	(i)	(3, 5)	<b>B1B1</b>	column vector <b>B0B1</b>
	(ii)	$m_{BD} \left( = \frac{6-4}{1-5} \right) = -\frac{1}{2}$ $m_{AC} \left( = -1 \div -\frac{1}{2} \right)$ seen or used $y - 5 = 2(x - 3)$ or $y = 2x + c$ , $c = -1$ or better	M1 M1 A1	can be implied by second M1
	(iii)	$p = 1$ $q = 7$ [ $A(1, 1)$ $C(4, 7)$ ] Method for finding area numerically  15	M1 M1  A1	could be in (ii) e.g. $24 - \left( \frac{1}{2} \times 1 \times 3 + \frac{1}{2} \times 1 \times 3 + \frac{1}{2} \times 4 \right)$ or shoelace method  <b>SC2</b> for 15 with no working
10	(i)	$-2 \sin 2x$ and $\frac{1}{3} \cos\left(\frac{x}{3}\right)$ Attempt at product rule $\frac{1}{3} \cos 2x \cos\left(\frac{x}{3}\right) - 2 \sin 2x \sin\left(\frac{x}{3}\right)$ isw	<b>B1+B1</b>  M1 A1ft	each trig function correctly differentiated  <b>ft</b> $k_1 \sin 2x$ and $k_2 \cos\left(\frac{x}{3}\right)$ provided $k_1, k_2$ are non-zero
	(ii)	$\sec^2 x$ and $\frac{1}{x}$ Attempt at quotient rule (with given quotient) $\frac{(\sec^2 x)(1 + \ln x) - \frac{1}{x}(\tan x)}{(1 + \ln x)^2}$ isw	<b>B1 + B1</b>  M1  A1	or rearrangement to correct product and attempt at product rule  penalise poor bracketing if not recovered

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11	(a)	$2^{x^2-5x} = 2^{-6}$ $x^2 - 5x + 6 = 0$ Correct method of solution of their 3 term quadratic $x = 2$ or $x = 3$	<b>M1</b> <b>M1</b> <b>M1</b> <b>A1</b>	Or $(x^2 - 5x)\ln 2 = \ln\left(\frac{1}{64}\right) = -6\ln 2$ <i>their "6"</i>
	(b)	Correct change of base to $\frac{\log_a 4}{\log_a 2a}$ $\frac{\log_a 4}{\log_a 2 + \log_a a}$ $\log_a a = 1$ used soi simplification to $\log_a 4$	<b>B1</b> <b>M1</b> <b>M1</b> <b>A1</b>	base $a$ only at this stage but can recover at end for $\log 2a = \log 2 + \log a$

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<p>12 (i)</p>	<p><math>f(3)</math> <math>\frac{6}{4}</math> oe</p>	<p><b>M1</b> <b>A1</b></p>	<p>or <math>fg(x) = \frac{2\sqrt{(x+1)}}{\sqrt{(x+1)+1}}</math></p>
<p>(ii)</p>	<p><math>2\left(\frac{2x}{x+1}\right)</math> <math>\frac{2x}{x+1} + 1</math></p> <p>A correct and valid step in simplification</p>	<p><b>M1</b></p>	<p>allow omission of <math>2(\dots)</math> in numerator or <math>(\dots) + 1</math> in denominator, but not both.</p>
<p>(iii)</p>	<p>Correctly simplified to <math>\frac{4x}{3x+1}</math></p> <p>Putting <math>y = g(x)</math>, changing subject to <math>x</math> and swapping <math>x</math> and <math>y</math> or vice versa</p> <p><math>g^{-1}(x) = x^2 - 1</math></p> <p>(Domain) <math>x &gt; 0</math> (Range) <math>g^{-1}(x) &gt; -1</math></p>	<p><b>dM1</b></p> <p>e.g. multiplying numerator and denominator by <math>x + 1</math>, or simplifying <math>\frac{2x}{x+1} + 1</math> to <math>\frac{2x+x+1}{x+1}</math></p> <p><b>A1</b></p>	<p>e.g. multiplying numerator and denominator by <math>x + 1</math>, or simplifying <math>\frac{2x}{x+1} + 1</math> to <math>\frac{2x+x+1}{x+1}</math></p>
<p>(iv)</p>		<p><b>M1</b></p> <p>condone <math>x = y^2 - 1</math>; reasonable attempt at correct method</p> <p><b>A1</b></p> <p>condone <math>y = \dots, f^{-1} = \dots</math></p> <p><b>B1</b></p> <p>condone <math>y &gt; -1 \quad f^{-1} &gt; -1</math></p>	<p>condone <math>x = y^2 - 1</math>; reasonable attempt at correct method</p> <p>condone <math>y = \dots, f^{-1} = \dots</math></p> <p>condone <math>y &gt; -1 \quad f^{-1} &gt; -1</math></p>
		<p><b>B1 + B1</b></p> <p>correct graphs; <math>-1</math> need not be labelled but could be implied by 'one square'</p> <p><b>B1</b></p> <p>idea of reflection or symmetry in line <math>y = x</math> must be stated.</p>	<p>correct graphs; <math>-1</math> need not be labelled but could be implied by 'one square'</p> <p>idea of reflection or symmetry in line <math>y = x</math> must be stated.</p>